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Optimal PMU Placement with Injection Measurements for Network Observability and State Estimation

Anubhav Goel[1] and Vivek Kumar[2]

[1,2]Electrical and Electronics Engineering Department, BRCMCET Bahal, Haryana
E-mail: [1]anubhavgoel207@gmail.com, [2]kvivek@brcm.edu.in

Abstract—*The most commonly used weighted least square state estimator in power industry is nonlinear and formulated by using conventional measurements such as line flow and injection measurement. The use of Phasor Measurement Units (PMUs) enhances the state estimation in terms of accuracy and fewer only are required for complete observability. The placement algorithm based on integer programming is used for the placement of PMUs in a network. Use of only PMUs for complete observability is not economical; in order to reduce the cost of installation placement of PMUs along with injection measurements in a conventional state estimation program will be discussed in this paper. The effect of adding injection measurements on number of PMUs used for network observability and also the state estimation solution accuracy will be studied. Case studies carried out on different size test systems are presented.*

Keywords*: Hybrid state estimation, network observability, phasor measurement units, state estimation.*

1. INTRODUCTION

Phasor Measurement Units (PMUs) using synchronization signals from the GPS satellite system have evolved into mature tools and are now being manufactured commercially [1]. As the PMUs become more and more affordable, their utilization will increase not only for substation applications but also at the control centres for the Energy Management Systems (EMS) applications. A PMU placed at a given bus is capable of measuring the voltage phasor of the bus as well as the phasor currents for all lines incident to that bus [2]. Hence, furnishing a selected subset of buses with PMUs can make the entire system observable. This will only be possible by proper placement of PMUs among the system buses. This problem is formulated and solved using graph theoretic observability analysis and an optimization method based on binary integer programming.

State estimation is a key element of the online security analysis function in modern power system energy control centres. The function of state estimation is to process a set of redundant measurements to obtain the best estimate of the current state of a power system. State estimation is traditionally solved by the weighted least square algorithm with conventional measurements such as voltage magnitude, real and reactive power injection, real and reactive power flow [3]. The voltage and current phasors obtained from PMUs can be implemented in the traditional state estimation and the effect of adding PMU measurements on the state estimation solution accuracy will be studied.

A specific model is used to implement both the voltage and line current phasor measurements into traditional WLS state estimation. In this model, the voltage phasor measurements are used in the polar coordinates denoted as the angle δ_i and magnitude V_i for the voltage phasor at the certain bus i, which directly corresponds to the state variables δ_i and V_i. The line current phasor are measured in rectangular coordinates, in terms of their real $I_{ij,(r)}$ and imaginary $I_{ij,(i)}$ parts for the current phasor in the branch from bus *i* to bus *j* [4].

In this paper optimal placement of PMUs along with injection measurements is discussed. Only use of PMUs for the observability of the system is highly accurate but at the same time it is very costly. From the economy point of view it is not desirable to install only PMUs for the observability of the system, so in order to minimize the cost of installation injection measurements are also placed along with the PMUs to make the system observable. Use of injection measurements minimizes the number of PMUs which reduces the cost of installation as well as the accuracy of the system is not affected by a large margin and it stays within the acceptable limit.

In order to implement optimum locations of injection measurements along with PMUs the objective function used for the optimum placement of PMUs is modified to include the injection measurements. Then binary integer programming is implemented using MATLAB to this new objective function to obtain the optimum locations.

2. WEIGHTED LEAST SQUARE STATE ESTIMATION METHOD

Weighted Least Square (WLS) method is commonly used to solve the state estimation problem, which is formulated as the following optimization problem:

$$\text{Minimize } \sum_{i=1}^{m} W_{ii} r_i^2 \quad (1)$$

Subjected to $z_i = h_i(x) + r_i$ i = 1,….., m

Where

m is the number of measurements; *n* is the number of system states;

$z^T = [z_1, z_2, \ldots\ldots, z_m]$ is the vector of measurement;

$h^T = [h_1(x), h_2(x), \ldots\ldots, h_m(x)]$ is a nonlinear measurement vector;

$x^T = [x_1, x_2, \ldots\ldots, x_m]$ is the system state vector.

W is the weight matrix, which is defined as the inverse of the covariance matrix of the measurement errors *R*:

$$R = \text{diag}\,[\,\sigma_1^2, \sigma_2^2, \ldots\ldots, \sigma_m^2] \quad (2)$$

At the minimum value of the objective function, the first order optimality conditions have to be satisfied.

These can be expressed in compact form as follows:

$$g(x) = -\frac{\partial J(x)}{\partial x} = -H^T(x)R^{-1}(z-h(x)) = 0 \quad (3)$$

Where

$$H(x) = \frac{\partial h(x)}{\partial x} \quad (4)$$

The nonlinear function *g*(*x*) can be expanded into its Taylor series around the state vector x^k neglecting the higher order terms. An iterative solution scheme known as the Gauss-Newton method is used to solve

$$x^{k+1} = x^k - [G(x^k)]^{-1}.g(x^k) \quad (5)$$

Where, k is the iteration index; x^k is the solution vector at the k_{th} iteration; $G(x^k)$ is called the gain matrix, and expressed by:

$$G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k)R^{-1}H(x^k) \quad (6)$$

$$g(x^k) = -H^T(x^k)R^{-1}[z-h(x^k)] \quad (7)$$

Substituting the values of equation (6) and (7) in equation (5) and solving we get:

$$[G(x^k)]\Delta x^{k+1} = H^T(x^k)R^{-1}[z-h(x^k)] \quad (8)$$

Where $\Delta x^{k+1} = x^{k+1} - x^k$

State vector x^k are calculated iteratively until the maximum variable difference satisfies the condition, ' $Max\ |\Delta x^k| < \varepsilon$ '. Consider a system having (*N*) buses; the state vector will have (*2N-1*) components which are composed of (*N*) bus voltage magnitudes and (*N-1*) phase angles.

The three most commonly used measurement used in state estimation are bus power injections, the line power flows and bus voltage magnitudes. These measurement equations can be expressed using the state variables. Jacobian matrix *H* has rows at each measurement and columns at each variable. *H* matrix components corresponding to these measurements are partial derivation of each variable.

$$H=\begin{bmatrix} \frac{\partial Pi}{\partial \delta} & \frac{\partial Pi}{\partial V} \\ \frac{\partial Pij}{\partial \delta} & \frac{\partial Pij}{\partial V} \\ \frac{\partial Qi}{\partial \delta} & \frac{\partial Qi}{\partial V} \\ \frac{\partial Qij}{\partial \delta} & \frac{\partial Qij}{\partial V} \\ 0 & \frac{\partial V_{Mag}}{\partial V} \end{bmatrix} \quad (9)$$

In this matrix δ and V are state variables, P_i and Q_i are real and reactive power injections at bus *i*. P_{ij} , Q_{ij} are real and reactive power flows from bus *i* to bus *j*.

3. HYBRID STATE ESTIMATION

In a hybrid state estimation the measurements received from the PMUs are incorporated in the traditional state estimation. One PMU can measure the voltage and the current phasors. The voltage phasor measurements are used in the polar coordinates denoted as the angle δ_i and magnitude V_i for the voltage phasor at bus *i*, which directly corresponds to the state variables δ_i and V_i. Therefore, there is a linear relation between the voltage phasor measurements and state variables. However, the model of line current phasor measurement is nonlinear and more complicated. The line current phasor are written in rectangular coordinates, in terms of their real $I_{ij,(r)}$ and imaginary $I_{ij,(i)}$ parts for the current phasor in the branch from bus *i* to bus *j*. Consider a two-port π −model of a network branch show in Fig. 1

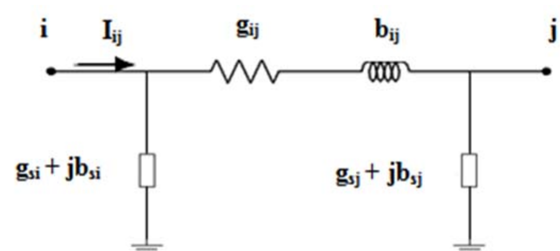

Fig. 1: π −Model of a Network Branch

where,

$g_{ij}+b_{ij}$ is the admittance of the series branch connecting buses *i* and *j* ;

$g_{si}+b_{si}$ is the admittance of the shunt branch connected at bus *i* .

The real and imaginary parts of the current phasor along the branch from bus *i* to bus *j* can be expressed as the following formulations, which also represent the nonlinear measurement functions h_i *(x)* relating current phasor measurements to the state variables:

$$I_{ij,(r)} = (V_i\cos\delta_i - V_j\cos\delta_j)g_{ij} - (V_i\sin\delta_i - V_j\sin\delta_j)b_{ij} + V_i\cos\delta_i\, g_{sh} - V_j\sin\delta_j b_{sh} \quad (10)$$

$$I_{ij,(i)} = (V_i\cos\delta_i - V_j\cos\delta_j)b_{ij} + (V_i\sin\delta_i - V_j\sin\delta_j)g_{ij} + V_i\cos\delta_i\, b_{sh} + V_j\sin\delta_j g_{sh} \quad (11)$$

Their corresponding elements in the Jacobian matrix *H* can also be obtained by the derivative of the real and imaginary with respect of the angle and voltage. The corresponding *H* matrix will become:

$$H = \begin{bmatrix} \frac{\partial Pi}{\partial \delta} & \frac{\partial Pi}{\partial V} \\ \frac{\partial Pij}{\partial \delta} & \frac{\partial Pij}{\partial V} \\ \frac{\partial Qi}{\partial \delta} & \frac{\partial Qi}{\partial V} \\ \frac{\partial Qij}{\partial \delta} & \frac{\partial Qij}{\partial V} \\ 0 & \frac{\partial V_{Mag}}{\partial V} \\ \frac{\partial \delta_{Mag}}{\partial \delta} & 0 \\ \frac{\partial Iij(r)}{\partial \delta} & \frac{\partial Iij(r)}{\partial V} \\ \frac{\partial Iij(i)}{\partial \delta} & \frac{\partial Iij(i)}{\partial V} \end{bmatrix} \quad (12)$$

4. PMU PLACEMENT PROBLEM FORMULATION

Case 1: A system which has only PMU measurements [2].

A PMU placed at a given bus is capable of measuring the voltage phasor of the bus as well as the phasor currents for all lines incident to that bus. Thus, the entire system can be made observable by placing PMUs at strategic buses in the system. The objective of the PMU placement problem is to accomplish this task by using a minimum number of PMUs [2].

For an n-bus system, the PMU placement problem can be formulated as follows:

$$\min \sum_{i=1}^{n} w_i\ x_i \quad (13)$$

$$\text{s.t. } f(X) \geq \hat{1}$$

where,

X is a binary decision variable vector, whose entries are defined as:

$$x_i = \begin{cases} 1 \text{ if PMU is installed at bus i} \\ 0 \text{ otherwise} \end{cases}$$

w is the cost of the PMU installed at bus i;

f (*X*) is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise.

$\hat{1}$ is a vector whose entries are all ones.

Constraint functions ensure full network observability while minimizing the total installation cost of the PMUs.

Consider the 5-bus system and its measurement configuration shown in Fig. 2.

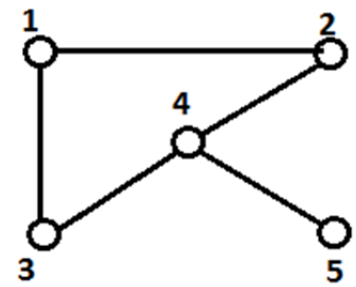

Fig. 2: 5-Bus Example System

First, form the binary connectivity matrix *A*. The entries of *A* are defined as follows:

$$A_{k,m} = \begin{cases} 1 \text{ if k} = \text{m} \\ 1 \text{ if k and m are connected} \\ 0 \text{ otherwise} \end{cases} \quad (14)$$

Matrix A can be directly obtained from the bus admittance matrix by transforming its entries into binary form. Building the A matrix for the 5-bus system of Fig. 2 yields

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (15)$$

The constraints for this case can be formed as:

$$f(X) = \begin{cases} f_1 = x_1 + x_2 + x_3 \geq 1 \\ f_2 = x_1 + x_2 + x_4 \geq 1 \\ f_3 = x_1 + x_3 + x_4 \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 \geq 1 \\ f_5 = x_4 + x_5 \geq 1 \end{cases} \quad (16)$$

The operator “+” serves as the logical “OR” and the use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in the sum will be non-zero.

The first constraint $f_1 \geq 1$ implies that at least one PMU must be placed at either of buses 1, 2 or 3 in order to make bus 1 observable. So after solving the constraints for this 5 bus system the PMU installed at bus 2 and 4 can make the whole system observable.

Case 2: A system which has injection measurements along with PMUs.

In this paper few injection measurements are also placed along with the PMUs to make the system observable and also to minimize the number of PMUs used in case 1.

In order to do this the objective function is modified and the effect of injection measurement is added to this function. For an n-bus system, the modified PMU placement problem can be formulated as follows:

$$\min \sum_{i=1}^{n} w_i \; x_i + w'_i \, x'_i \qquad (17)$$

s.t. $f(X) \geq \hat{1}$

where

X is a binary decision variable vector, whose entries are defined as:

$$x_i = \begin{cases} 1 \text{ if PMU is installed at bus i} \\ 0 \;\; \text{otherwise} \end{cases}$$

$$x'_i = \begin{cases} 1 \text{ if injection measurement is at bus i} \\ 0 \;\; \text{otherwise} \end{cases}$$

w is the cost of the PMU installed at bus *i*;

w' is the cost of the injection measurement at bus *i*;

The cost of PMU is considered to be twice than the cost of injection measurement so the value of *w* is taken as 2 and the value of *w'* is taken as 1.

f (*X*) is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise.

$\hat{1}$ is a vector whose entries are all ones.

Constraint functions ensure full network observability while minimizing the total installation cost of the PMUs with injection measurement.

Consider the 5-bus system and its measurement configuration shown in Fig. 2.

First, form the connectivity matrix *A*. The entries of *A* are defined as follows:

$$A_{k,m} = \begin{cases} 2 \text{ if } k = m \text{ for PMU} \\ 2 \text{ if k and m are connected for PMU} \\ 1 \text{ if } k + n = m \text{ for injection measurement} \\ 0 \text{ otherwise} \end{cases}$$

Building the *A* matrix for the 5-bus system of Fig. 2 yields

$$A = \begin{bmatrix} 2 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (18)$$

The constraints for this case can be formed as:

$$f(X) = \begin{cases} f_1 = 2x_1 + 2x_2 + 2x_3 + x'_1 \;\geq 1 \\ f_2 = 2x_1 + 2x_2 + 2x_4 + x'_2 \;\geq 1 \\ f_3 = 2x_1 + 2x_3 + 2x_4 + x'_3 \;\geq 1 \\ f_4 = 2x_2 + 2x_3 + 2x_4 + 2x_5 + x'_4 \geq 1 \\ f_5 = 2x_4 + 2x_5 + x'_5 \;\geq 1 \end{cases} \qquad (19)$$

The operator "+" serves as the logical "OR" and the use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in the sum will be non-zero.

After solving the constraints for this 5 bus system the PMU installed at bus 4 and injection measurement at bus 1 can make the whole system observable.

Case 3: Placement strategy against loss of a single PMU or injection measurement.

So far it is assumed that those PMUs and injection measurement which are placed by the proposed method will function perfectly. But, they are prone to failure just like any other measuring device. In order to guard against such unexpected failures, the above placement strategy is extended to account for single PMU or injection measurement loss. In this study, this objective is achieved by choosing two independent sets, a primary set and a backup set, each of which can make the system observable on its own. If any PMU or injection measurement is lost, the other set of PMUs or injection measurement will guarantee the observability of the system.

The backup and primary set of PMUs is chosen by building the constraint functions according to the procedures described in previous subsections with the only difference of change of right hand side $\hat{1}$ to $\hat{2}$.

5. SIMULATION RESULTS

Simulations are carried out on the IEEE 14-bus system. Binary Integer programming problem is solved using the MATLAB for the PMU placement problem formulation. In addition a program written in MATLAB is used for state estimation including the measurements from PMUs.

5.1 PMU Placement

IEEE 14-bus system used for simulation is shown in Fig. 3. Table 1 shows the results for 14-bus system without considering any PMU or injection measurement loss and Table 2 shows the results for 14-bus system considering single PMU or injection measurement loss.

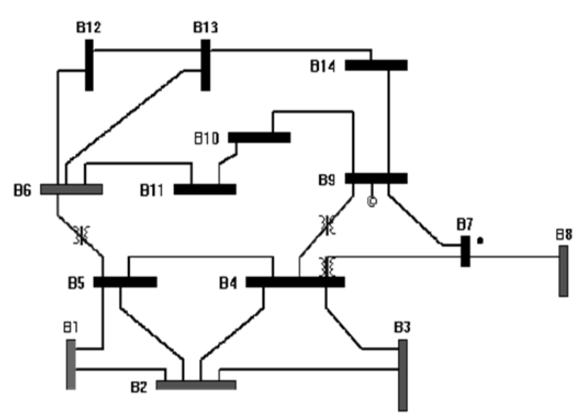

Fig. 3: IEEE 14-Bus System

TABLE I: Results for 14-bus system without Any PMU or injection measurement loss

ONLY PMU		PMU WITH INJECTIONS	
NO. OF PMU	LOCATION (BUS)	NO. OF PMU	LOCATION (BUS)
4	2,6,7,9	3	2,6,9
		NO. OF INJECTION	LOCATION (BUS)
		1	8

TABLE II: Results for 14-bus system considering single PMU or injection measurement loss

ONLY PMU(BACKUP)		PMU WITH INJECTIONS(BACKUP)	
NO. OF PMU	LOCATION (BUS)	NO. OF PMU	LOCATION(BUS)
9	2,3,5,6,7,8,9,11,13	6	2,6,7,9,11,13
		NO. OF INJECTION	LOCATION(BUS)
		3	1,2,8

5.2 Cost comparison in both cases

Since the cost of PMU is assumed to be twice of a injection measurement the cost function can be calculated as follows

Cost = 2 × no. of PMUs + 1 × no. of injection measurement

Let the cost of one Injection measurement device be x then the cost of PMU will be 2x.

Table 3 shows the comparison of cost for 14 bus system.

TABLE IIIIIIV: Cost comparison for 14-bus system

	COST
ONLY PMU	8x
PMU WITH INJECTIONS	7x
ONLY PMU (BACKUP)	18x
PMU WITH INJECTIONS (BACKUP)	15x

From this table it can be seen that the cost of PMU with injection is less than using only PMU while the network is still observable. So the method to use PMU along with injection measurement is more economical than using only PMU.

5.3 Comparison of estimation accuracy in both cases.

To investigate the accuracy of estimated variables, both the cases are tested with only PMUs and PMUs with injection measurement. Test system (IEEE 14 bus system) is tested with 2 different cases namely

1) Only Minimum PMUs

2) Minimum PMUs with Injection Measurements.

Fig. 3 shows the network diagram for the system. A network has a voltage magnitude measurement connected to bus 1.

The setting of error standard deviations for power injection is taken as 0.01. A PMU has much smaller error deviation than other conventional measurements and is taken as 0.00001in this study. The different measurement parameters used for state estimation of 14 bus system and 30 bus system are taken from [7] and [8] respectively.

One of the ways of representing the level of state estimation accuracy is to refer the covariance of the estimated variables. The variances of variables are obtained from the inverse diagonal elements of gain matrix. The accuracy of two variables (voltage magnitude and voltage angle) is investigated separately. Fig. 4 shows the accuracy of the estimated voltage magnitudes of two systems IEEE 14 bus system. Fig. 5 shows the accuracy of the estimated voltage angles of IEEE 14 bus system.

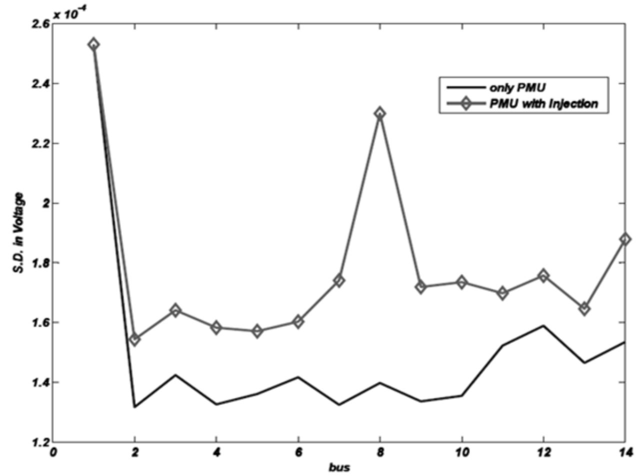

Fig. 4: Accuracy of Voltage Estimates for Two Cases in IEEE-14 Bus System

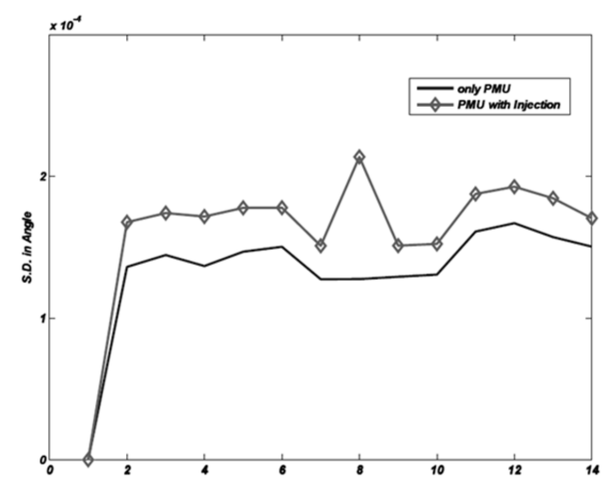

Fig. 5: Accuracy of Voltage Angle Estimates for Two Cases in IEEE-14 Bus System

This clearly shows that the accuracy of the system having PMUs with injection measurement is less than the accuracy of system having only PMU , the difference is very small and can be neglected, thus the accuracy in both the cases can be taken as almost equal.

6. CONCLUSION

In this paper, two types of placement techniques of PMU are discussed for state estimation, one which contains only PMUs and other which contains both PMUs and injection measurement. The optimum locations for placement of only PMUs and PMUs along with injection measurement are found for network observability with and without considering loss of singe PMU or injection measurement. Both the techniques are tested on IEEE 14 bus system. Binary Integer Programming is done on MATLAB. The optimum locations obtained from the results are utilized for the calculation of cost of installation and covariance of the estimated variables. Their benefits to state estimation are studied with respect to cost and accuracy. The optimal placement of PMUs with injection measurements is more economical and the estimates obtained are of almost same accuracy as compared to the optimal placement of only PMUs.

REFERENCES

[1] Bei Gou “Generalized Integer Linear Programming Formulation for Optimal PMU Placement” IEEE Transactions on Power Systems, Vol. 23, No. 3, pp. 1099-1104, August 2008.

[2] B. Xu and A. Abur, “Observability Analysis and Measurement Placement for Systems with PMUs,” in Power Systems Conference and Exposition, 2004. IEEE PES, pp. 943 – 946 vol.2, 10-13 Oct. 2004.

[3] A.G. Phadke, J. S. Thorp, and K. J. Karimi, “State Estimation with Phasor Measurements”, IEEE Transactions on Power Systems, Vol. 1, No. 1, pp. 233-241, February 1986.

[4] R. Zivanovic, C. Cairns, “Implementation of PMU Technology in State Estimation: An Overview”, IEEE Conferences, Sep 1996 , Page(s): 1006 - 1011 vol.2

[5] DF A. G. Phadke, “Synchronized phasor measurements in power systems”, IEEE Computer Applications in Power, Vol. 6, Issue 2, pp. 10-15, April 1993.

[6] D K. Zhu, L. Nordstrom, “Application and analysis of Optimum PMU Placement methods with application to State Estimation Accuracy” PES '09. IEEE _2009, Page(s): 1 – 7.

[7] Satyendra. P. Singh, S. P. Singh, “Optimal PMU Placement in Power System Considering the Measurement Redundancy”, Int. J. of Advances in Electronic and Electric Engineering, Vol. 4, No. 6, Jan. 2014, pp. 593-598.

[8] http://www.ee.washington.edu/research/pstca/pf14/ieee14cdf.txt (accessed on June 2011)